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RADIATION FROM FLAMES AND GASES IN A COLD WALL COMBUSTION CHAMBER

B. LECKNER
Department of Steam Engineering, Chalmers University of Technology, 402 20 Göteborg 5, Sweden

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Abstract—The calculation of heat transfer in furnaces is usually based on coarse approximations of temperature and emissivity in the combustion chamber. This paper suggests a method suitable for machine computation of the heat transfer from flames and gases to the walls of a furnace. A theoretical derivation of the basic equation is made and the simplifications necessary for execution of the calculation in acceptable time are discussed. The patterns of temperature and flame emissivity are described by means of simple functions, a mathematical model. Experiments were carried out in an experimental furnace of simple geometry using one oil-burner in order to check the accuracy of the theoretical assumptions. Measurements of flame and gas temperature, flame emissivity, wall heat flux and exterior measurements necessary for the heat balance of the furnace were done. The measurements of temperature and emissivity define the mathematical model.

The heat transfer from flame and gases (including the contribution due to reflexion, non-cold surfaces and convection) was calculated at four different loads of the furnace and compared with the heat flux measurements. The difference between measured and calculated heat flux, due to errors of measurements and calculations, was less than about 15 per cent of the total heat flux.

NOMENCLATURE

A,	area of surface;
A_K, A_T,	constants, equations (16) and (12);
B_K, B_T,	constants, equations (20) and (19);
b_o,	halfwidth of the combustion chamber;
c,	constant;
D,	side of an element;
E_b,	black body emissive power;
I,	intensity of radiation;
K_a,	coefficient of absorption;
K,	coefficient of emission, coefficient of extinction;
L,	length or characteristic dimension;
Q,	energy received by the radiation pyrometer;
q,	hemispherical radiative flux;
R,	distance from the emitting element to the receiving element;
ΔR,	steplength along R;
r,	radius;
$\mathbf{r}$,	radius vector;
$\mathbf{s}$,	unit vector;
s, s_0, s_1,	points on a line in the direction of $\mathbf{s}$;
T,	temperature;
V,	volume;
x, y, z,	coordinates;
$\Delta x, \Delta y, \Delta z$,	side of the elements;

Greek symbols

α,	half of the jet angle;
ε,	emissivity;
ρ,	reflectance;
η,	energy emitted from a volume element;
θ,	angle of incidence;
λ,	wavelength;
λ,	constant in equations (16) and (17);
μ,	$\cos\theta$;
σ,	Stephan–Boltzmann constant $5{\cdot}669 \times 10^{-8}$ W/m^2 °K^4;
τ,	optical depth;

Φ, 'escape factor';
Ω, solid angle;

Subscripts
i, j, l, volume elements;
A, surface elements;
M, adiabatic surface;
N, along R;
b, black body;
λ, wavelength;
g, gas.

INTRODUCTION

In order to make an exact calculation of the radiation from flames and gases in a furnace it is necessary to know the temperature, the distribution of emitting and absorbing material and its emitting and absorbing capacity. It is possible to derive the theoretical equations describing combustion, flow and energy distributions in the space of the combustion chamber, but it is impossible to solve them, for instance, the coupling between the velocity field and the energy field (where the radiation is included) gives a non-linear integro-differential equation in three dimensions. Besides, the constants included in the different expressions are not always well known.

Without solving the above mentioned equations it could however be possible to draw certain conclusions about the functioning of the furnace by means of the similarity theory. Such work has been done mainly in the Soviet Union (Konakov [1], Gurvich [2]), but also by others, e.g. [3, 4]. However, the number of dimensionless relations thus obtained is so great that coarse simplifications must be done in order to draw any practical conclusions from them. Usually the analysis ends up with a relation between two temperatures, where the reference temperature is the adiabatic flame temperature or the gas temperature at the exit of the combustion chamber, and a dimensionless number containing some quantities defining the process. This is the outlook of most of the equations reviewed in [5]. Mean-values of temperature and emissivity are used to describe the radiation heat transfer. Sometimes a limited approach is made and only the flow of gases in a certain type of furnace is described in terms of dimensionless numbers [6, 7] and from these, conclusions are drawn about heat transfer.

Hottel and co-workers [8, 9] calculated the heat flux distribution and the temperature field in gas-fired cylindrical furnaces with homogeneous gas composition for different assumed flow patterns and burning rates. They divided the volume of the combustion chamber and the surrounding surfaces into zones (38 zones) and made a heat balance on each zone. By solving the non-linear simultaneous equations resulting, the temperature of each zone was obtained. These temperatures were used to calculate the wall heat flux distribution.

Günther and Hering [10] have tried to use the method mentioned above, but they have not reached a result, the accuracy of which is compatible with the effort of using it, mainly on account of insufficient knowledge of the progress of combustion and the distribution of the combustion products in space. In the present calculation, concerning luminous flames these difficulties remain the same and moreover the local emissivity of luminous, turbulent diffusion flame is little known. If a sufficiently accurate calculation of the burning rate and the distribution of combustion products could be made, the temperature field and the distribution of emissivity could be obtained. At present, however, it seems to be an easier and more direct approach for radiation calculations to assume a temperature and an emissivity pattern from the general outlook of the burner arrangement and the geometry of the furnace.

THEORETICAL BASIS

The radiative transfer in the direction of a vector **s** within the element of solid angle $d\Omega$ may be written for an absorbing and emitting medium

$$\frac{dI}{ds} = \eta - K_a I \qquad (1)$$

where I is the intensity of radiation,

K_a is the coefficient of absorption,

η is emitted energy from an element confined by ds and $d\Omega$.

The integration of equation (1) in the direction of $\mathbf{s}$ from s_0 to s gives the intensity in s

$$I(s) = I(s_0) \exp[-\tau(s_0, s)] + \int_{s_0}^{s} \eta(s_1) \exp[-\tau(s_1, s)]\, ds_1 \qquad (2)$$

where $s_0 \leqslant s_1 \leqslant s$

$$\text{and} \quad \tau(s_0, s) = \int_{s_0}^{s} K_a ds.$$

In a furnace the absorbing and emitting medium is confined within the walls and the radiative flux from the medium and from the walls to a point s on the wall is

$$q(s) = \int_{\Omega = 2\pi} I(s) \cos\theta\, d\Omega \qquad (3)$$

where θ is the angle of incidence of a beam of radiation, and the position coordinate is described by a radius vector $\mathbf{r}$, and

$$\cos\theta\, d\Omega = \frac{\cos\theta \cos\theta_0}{|\mathbf{r} - \mathbf{r}_0|^2} dA'$$

$$\cos\theta\, ds_1\, d\Omega = \frac{\cos\theta\, dV'}{|\mathbf{r} - \mathbf{r}_1|^2}$$

where dA' is a surface element on the bounding surface A

dV' is a volume element of the volume of the furnace V

θ_0 is the angle by which the radiation leaves the emitting surface.

The total irradiation on a surface element on the wall is given by equations (2) and (3)

$$q(\mathbf{r}) = \int_A I(\mathbf{r}_0) \exp[-\tau(\mathbf{r}_0, \mathbf{r})] \frac{\cos\theta \cos\theta_0}{|\mathbf{r} - \mathbf{r}_0|^2} dA' + \int_V \eta(\mathbf{r}_1) \exp[-\tau(\mathbf{r}_1, \mathbf{r})] \frac{\cos\theta\, dV'}{|\mathbf{r} - \mathbf{r}_1|^2} \qquad (4)$$

here

$$I(\mathbf{r}_0) = \frac{q^+(\mathbf{r}_0)}{\pi}$$

where q^+ is the total flux leaving the surface element

$$q^+ = \varepsilon E_b + \rho q \qquad (5)$$

where ε is the emissivity of the surface

$\rho = 1 - \varepsilon$ is the reflectance of the surface

q is, as above, incident radiation

E_b is the black body emissive power of the surface.

Equation (4) is the integral equation of radiative transfer which is to be used under certain assumptions in the following calculation.

APPROXIMATIONS AND ASSUMPTIONS

In order to solve equation (4) without too much computational effort certain simplifications will be introduced.

1. The enclosure of the furnace will be assumed to consist of a finite number of adiabatic surfaces, e.g. refractory surfaces and of water cooled furnace walls which are black and cold.

At an adiabatic surface, the flux incident on the surface must be equal to the flux leaving the surface

$$q^+ = q.$$

Then, according to equation (5), $q^+ = E_b$ in this case. As the cold part of A does not contribute in the integration, $I(\mathbf{r}_0)$ in the first integral of equation (4) becomes equal to E_b/π and the equation is no longer an integral equation, but a sum of integrals. Errors due to the cold wall approximation are negligible in a water-cooled furnace, whereas the deviation of wall emissivity from unity might influence the result of the calculations. (This will be discussed in a later section.)

2. Kirchhoff's law is valid. Then $\eta = KI_b$ [or $\eta = K(E_b/\pi)$] and $K = K_a$. K is the coefficient of emission.

This is not exactly correct when radiation emitted from a volume element with the temperature T_1 is absorbed in another volume element with the temperature T_2 if $T_1 \neq T_2$. However, the radiation emitted from a volume element of high capacity of emission is mostly absorbed by neighbouring elements with high capacity of absorption and the temperature difference between these elements is not so large (due to the similar character of the fields of emittive capacity and temperature).

3. The radiation from flames and gases is assumed to be grey. This is an important simplifying approximation. It can be made due to the fact that most of the radiative transfer occurs in the luminous flame, which consists mainly of soot particles of the size of about 0·01–0·08 μm [11]. In this case the scattering may be neglected [12]. The luminous flame is surrounded by gases consisting mainly of CO_2, H_2O and N_2. The assumption of a grey flame is valid as long as the gases are treated as grey gases. In the case of gases, the calculation of radiative transfer is dependent on the correct choice of a mean beam length. This choice is rather approximate as the form of the gas envelope is irregular. However, the contribution from the gas-filled parts is small compared to that from the luminous part of the flame, and an estimate of the mean beam length valid in the largest gas-filled space will do. A grey-clear gas approximation according to Hottel [9] would give a better description of gas emissivities, but cannot fit into the equations theoretically derived. The above equations are derived for monochromatic radiation although the index λ (wavelength) has been omitted. The equations will be the same for quantities integrated over the whole spectrum for grey radiation if a suitable mean-value for K can be found. If K_λ is constant along ds, the monochromatic emissivity is defined by

$$\varepsilon_\lambda = 1 - \exp(-K_\lambda \, \mathrm{d}s) \tag{6}$$

and the grey emissivity will be

$$\varepsilon = \frac{\int_0^\infty (1 - \exp(-K_\lambda \, \mathrm{d}s)) I_{b\lambda} \, \mathrm{d}\lambda}{\int_0^\infty I_{b\lambda} \, \mathrm{d}\lambda} \tag{7}$$

K_λ is wavelength dependent in luminous flames and varies according to $K_\lambda = c\lambda^{-n}$ where c is a constant and n is close to unity [11]. Equation (7) has been solved graphically and it yields practically the same result as is obtained by substituting the emission mean wavelength at temperature T (emission mean wavelength [13] is defined by $\lambda_{0\cdot 5 E_b} \cdot T = 0{\cdot}411$ cm °K) in equation (6) [11].

Thus, for grey medium

$$\varepsilon = 1 - \exp(-K \mathrm{d}s)$$

and K is a function of temperature and concentration of the radiating medium. This is an energy-weighted mean emisssivity and it will be applied also for gases.

4. Applying these assumptions, the integrals in equation (4) can be expressed in form of sums of finite surface and volume elements. As most combustion chambers have a rectangular shape, a Cartesian system of coordinates x, y, z will be used. Each volume element of the size of $D^3 = \Delta x \cdot \Delta y \cdot \Delta z$ and each surface element D^2 will be assigned a temperature and a value of K or ε, and the element will be considered homogeneous. This is a form of solution which is easy to carry out on a digital computer and the time of computation can be weighted against the accuracy requirements of the calculation.

Owing to the finite size of the volume elements, self-absorption in an element has to be accounted for. This is done by means of an 'escape-factor' that expresses the relation between the radiation leaving a homogeneous element and the radiation produced in the element. The calculation of this coefficient Φ has been performed by Hottel and Cohen [8] and the result is shown in Fig. 1. Equation (4) can now be written

$$q(x_A, y_A, z_A) = \sum_{M} \sum_{j_M} \sum_{l_M} E_{bM} \times \exp\left[-\sum_{N_M} K_{N_M} \Delta R\right]$$

$$\times \frac{|x_A - x_M| \cdot |z_{l_M} - z_A|}{\pi R^4} \Delta y \, \Delta z$$

$$+ \sum_i \sum_j \sum_l \Phi \cdot K(x_i, y_j, z_l)$$

$$\times E_b(x_i, y_j, z_l) \cdot \exp\left[-\sum_N K_N \Delta R\right]$$

$$\times \frac{|z_l - z_A|}{\pi R^3} \Delta x \, \Delta y \, \Delta z \qquad (8)$$

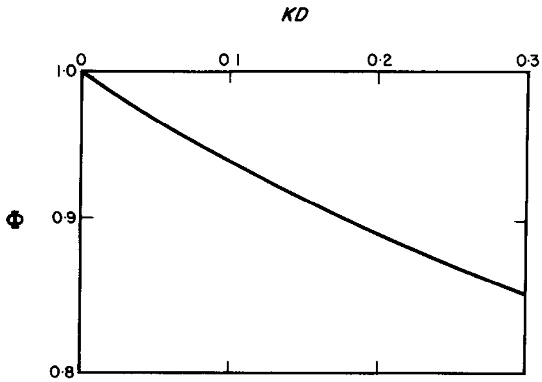


FIG. 1. Escape factor Φ for emission from a cube of edge D filled with medium of an absorption coefficient K [8].

Here

$E_b = \sigma T^4$

σ = Stephan–Boltzmann constant, $5{\cdot}669 \times 10^{-8}$ W/m², °K⁴

M = number of adiabatic surfaces

i, j and l = number of Δx, Δy, Δz

$R = |\mathbf{r} - \mathbf{r}_1|$ or $R = |\mathbf{r} - \mathbf{r}_0|$

$R^2 = (x_A - x_i)^2 + (y_A - y_j)^2 + (z_A - z_l)^2$

x_A, y_A, z_A = coordinates of the receiving element

$\tau(\mathbf{r}_1, \mathbf{r}) = \sum_N K_N \Delta R$

ΔR is the step-length along R

N is the number of ΔR along R, $N = R/\Delta R$

$$\cos\theta = \frac{|z_1 - z_A|}{R}$$

$$\cos\theta_0 = \frac{|x_A - x_M|}{R}.$$

In equation (8) the surfaces M are perpendicular to the side walls (zy-surfaces) and situated at x_M. The surfaces could of course have any position and the equation is easily modified according to the requirements.

5. The heat flux q in any point x_A, y_A, z_A on the wall of the furnace can now be calculated if the temperature of the refractory surfaces and the functions

$$K = K(x, y, z) \qquad (9)$$

$$T = T(x, y, z) \qquad (10)$$

are known.

The shape of the functions (9) and (10) depend on many factors characteristic for the furnace performance, ultimately, flow pattern, mixing rate, combustion rate and distribution of combustion products. The easiest way of defining the functions would be to assume constant mean values K = const and T = const which are valid over the whole combustion chamber as was discussed in the introduction. This would, however, mean an unnecessarily coarse approximation.

A one-dimensional variation along an axis situated in a suitable way in the combustion chamber would be a better approximation.

$$K_0 = \text{const} \cdot f_1(x) \qquad (11)$$

$$T_0 = \text{const} \cdot g_1(x) \qquad (12)$$

Here const means some constant to be defined, and $f(x)$ and $g(x)$ are functions of x.

An even better description of the actual conditions in a combustion chamber would be obtained by a function which accounts for the variation of the quantities T and K also in planes perpendicular to the x-axis

$$K = K_0 \cdot f(y, z)$$

$$T = T_0 \cdot g(y, z)$$

These functions might also depend on x, which is better to write

$$K = K_0 \cdot f_2(x, y, z)$$

$$T = T_0 \cdot g_2(x, y, z)$$

In this way the fields of T and K in the space limited by the walls of the combustion chamber can be written

$$K = \text{const}_K \cdot f_1(x) \cdot f_2(x, y, z) \quad (13)$$

$$T = \text{const}_T \cdot g_1(x) \cdot g_2(x, y, z). \quad (14)$$

The equations (13) and (14) together with equation (8) now form a mathematical model which can be used for calculations of radiative transfer in a combustion chamber, preferably filled mainly by luminous flames. The accuracy of the model depends on how well the functions f and g with respective constants will describe the real T and K patterns in the combustion chamber, provided that the above assumptions do not seriously influence the accuracy of the calculations.

The x-axis will be positioned in the direction of the flames, for instance coinciding with the axis of the central flame or in some other suitable symmetry position and thus the f_1 and g_1 functions represent the behaviour of the flame properties from the burner along the axis of the flame. The f_2 and g_2 functions represent the distribution of flame properties across the flames and are approximatively repetitive in the case of many flames. f_1 and g_1 could be straight lines or exponential series (one or two terms will be sufficient) and the f_2 and g_2 could be simple exponentials or sine-functions. The type of function depends on the type of burner and burner arrangement. A detailed definition of the functions may be made by defining the constants included in the functions by assumption and/or by experiment. Further, the best approach to the problem would be to define dimensionless functions so that experiments could be extended by means of similarity laws.

EXPERIMENTAL VERIFICATION

An experimental furnace of simple geometry, equipped with one burner and with a combustion chamber which is accessible to measurements, is a suitable arrangement for trials in order to verify the reliability of the above assumptions and derivations. As these experiments were carried out to check the accuracy of the theoretical derivations, quite a lot of measurements were made, the only simplifying assumption, except those accounted for above, being that of symmetry around the flame axis (coinciding with the centre line of the combustion chamber).

Experimental arrangements

The experimental furnace consists of a horizontal water-cooled chamber of square cross-section and a length:side ratio of 6:1. On the side walls there are observation openings. The burner is mounted in the centre of one of the end surfaces, where a refractory material is applied (Fig. 2).

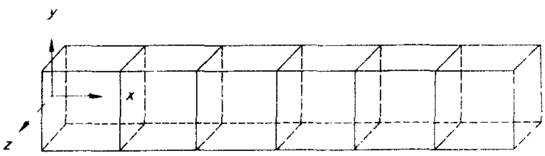


FIG. 2. Experimental furnace.

The experiments were run at four heat input rates: 110, 84, 60 and 47 l oil/h. These runs are called 1A, 2A, 3A, 4A and similar runs B, C and D were made to investigate the repeatability. The working conditions of the furnace (except the heat input rate) were kept as constant as possible during the four runs and measurements enabling heat-balance calculations for the complete furnace were done. The temperature field and the heat-flux on one side-wall were measured with calibrated suction pyrometers and calibrated heat-flux meters of the conduction plug type respectively. The integrated extinction coefficient K could be measured in the same points as the temperatures by means of a sensitive vacuum thermopile, mounted in a device which was introduced into the flame, thus registering the emission of a layer of the flame thin enough to be considered homogeneous and large enough not to be influenced by the apparatus (in this case 45 mm). The thermopile was kept at a constant temperature by water-cooling and the flame layer was limited on the side of the thermopile by nitrogen, flushed through the instrument and sucked out through the orifice which defines the beam of radiation to the receiving surface of the thermopile. The opposite side of the flame layer was limited either by a hot background, the temperature of which could be measured, or by a cold black-body background. In the later case the flame was prevented from entering the background by means of nitrogen filling the background cone and being sucked out at the limiting orifice of the background (Fig. 3).

where Q is measured by the instrument and the constant depends on the geometry and calibration of the instrument. E_b is obtained from the temperature measurements. The term $[1 - \exp(-KD)]$ is the emissivity of a flame layer of the thickness D.

Results of the measurements

The results of the measurements which will be used to define the functions (13) and (14) are presented in Fig. 4, and the heat flux measurements which are used to check the calculations are plotted in the diagrams of Figs. 10 and 11.

Choice of a model

Generally, the axial variation of flame properties (11) and (12) can be represented by exponential functions

$$T_0 = A_T(e^{-\lambda_{11}x/L} - e^{-\lambda_{12}x/L}) \qquad (16)$$

$$K_0 = A_K(e^{-\lambda_{21}x/L} - e^{-\lambda_{22}x/L}) \qquad (17)$$

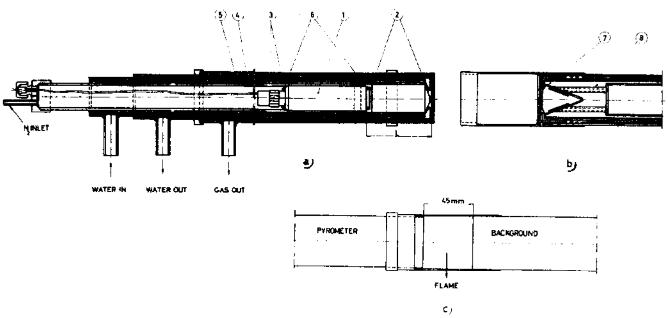


FIG. 3. Radiation pyrometer with cold black-body background.
(a) Radiation pyrometer. 1. Vacuum thermopile. 2. Orifices. 3. Channels for cooling-water. 4. External sheath of the gas channel. 5 Holder for the thermopile. 6. Channels for N_2. (b) Background, water cooled and supplied with N_2 in the same manner as the pyrometer, introduced into the flame from the opposite side of the funace. 7. Background cone. 8. Gas channels. (c) pyrometer and background in working position.

Experiments have shown the cold black-body instrument to be the most reliable and this instrument was used during the trials presented here. K is obtained from an equation derived by integrating equation (1) and the result is

$$Q = \text{const}\,(1 - e^{-KD})E_b \qquad (15)$$

L is a characteristic length (the length of the furnace) and A_T, A_K and λ are constants to be defined.

The second term determines the value close to the burner and could be omitted in the present type of flame where the narrow part of the flame close to the burner gives a rather small contri-

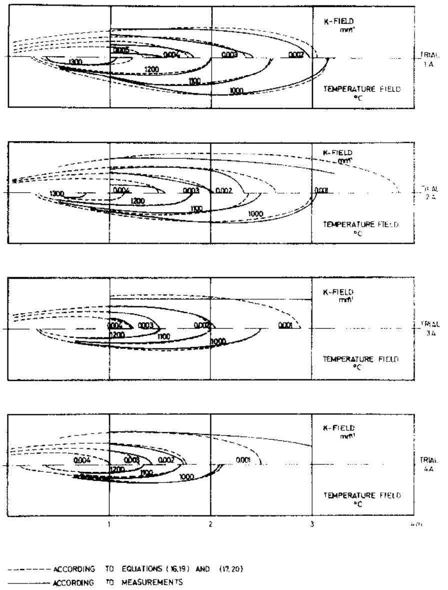


FIG. 4. The distribution of temperature and emissivity in a horizontal plane through the centre line of the furnace. (The first four sections of the furnace are shown.)

bution to the heat flux on the wall. However, to show the influence of the term, it will be retained in the T_0 function. The K-field is not very well known close to the burner, where the soot particles are to a great extent mixed with oil drops which are not completely burnt, and

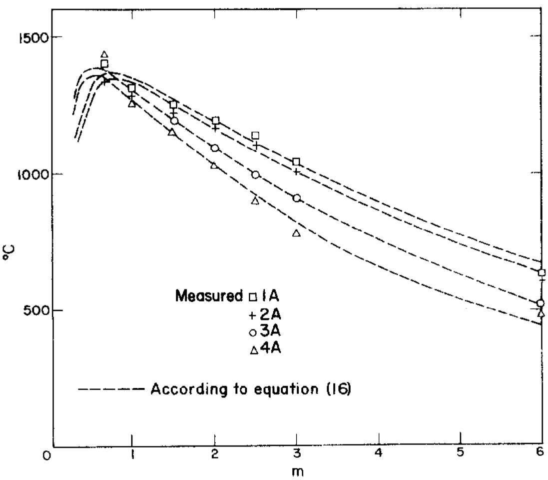


FIG. 5. Temperature along the flame axis (x-axis).

readings are difficult. Owing to the above mentioned facts, in equation (17) the second term may as well be omitted. With a suitable selection of constants A_T, A_K and λ the corresponding curves will give a good approximation of the measured points, Figs. 5 and 6. The constants are compiled in Table 1. The maximum and minimum values of the fields will be discussed below.

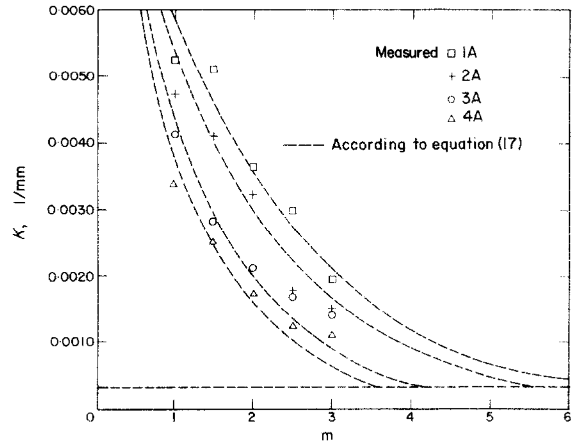


FIG. 6. Absorption coefficient along the flame axis (x-axis).

In a free turbulent jet that issues from a nozzle into the open air the lateral temperature and concentration profiles of the jet medium assumes exponential forms which are similar over the whole jet if expressed in a dimensionless form [14]. According to [15] a similar distribution may approximately be continued even if the jet is confined by walls. This cannot of course be immediately applied to a turbulent diffusion flame issued from a burner of a certain design in a confined space of a certain geometry. The burner gives a distribution of the fuel sprayed into the combustion chamber, which depends on the construction of the burner, and which is certainly different from that of a free jet. The walls of the combustion chamber disturb the similarity of the lateral distributions and limit the amount of air available to diffuse into the jet, thus causing recirculation.

However, for a large class of burners an exponential distribution will provide a sufficiently accurate approximation for heat transfer calculations.
Assume

$$\frac{T}{T_0} = \frac{K}{K_0} = e^{-(r/b)^2}$$

where $r = (y^2 + z^2)^{\frac{1}{2}}$ is a variable radius $0 \leqslant r \leqslant b_0$ in Fig. 7.
$b = x \cdot tg\alpha$ and $0 \leqslant b \leqslant b_0$ and α is half of the jet-angle.

$$\frac{T}{T_0} = \frac{K}{K_0} = e^{-B(y^2+z^2/x^2)} \quad (18)$$

$$B = \frac{1}{tg^2\alpha}$$

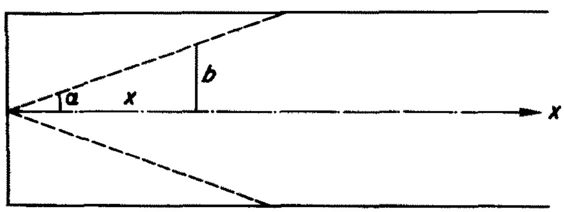


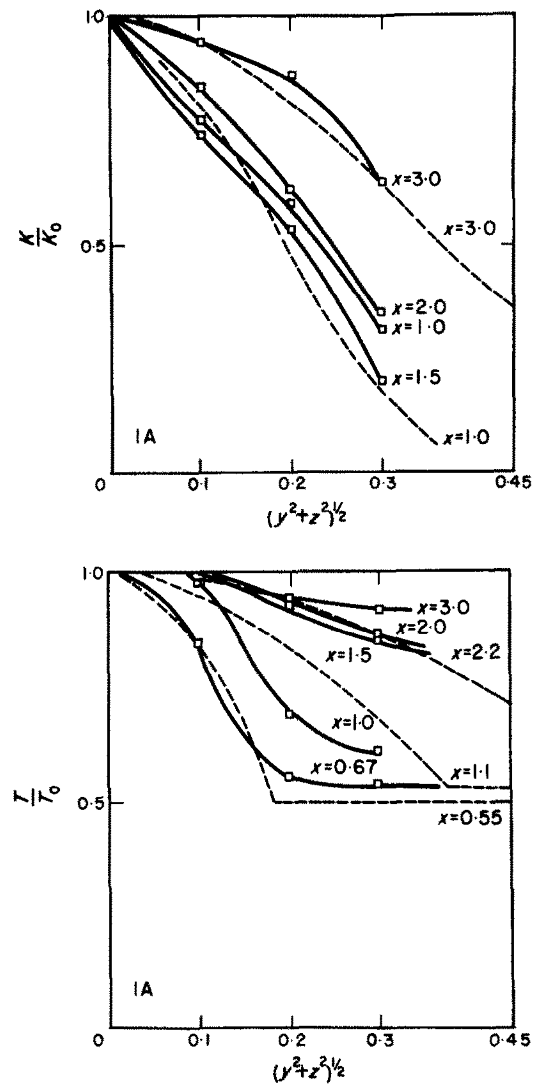
FIG. 7. Schematic representation of the jet.

In Figs. 8 and 9 equation (18) is fit to the measured values and it is seen that the best fit is

$$\frac{T}{T_0} = \exp -B_T \frac{y^2 + z^2}{x^2} \quad (19)$$

$$\frac{K}{K_0} = \exp -B_K \frac{y^2 + z^2}{(x + 1000)^2} \quad (20)$$

Note that the K-field is broader than the temperature field. The temperature and K-field are shown with dotted lines in Fig. 4.



FIGS. 8 and 9. Lateral distribution of temperature and absorption coefficient. K/K_0 and T/T_0 are represented as functions of $(y^2 + z^2)^{\frac{1}{2}}$ for different x. The points are measured values. The broken lines represents equations (19) and (20).

Table 1.

Trial	A_T	λ_{11}	λ_{12}	B_T	A_K	λ_{21}	B_K
1A	1570	0·85	30	6·0	0·01	3·1	71·9
2A	1570	0·90	30	9·7	0·01	3·7	71·9
3A	1570	1·10	42	11·1	0·01	4·8	71·9
4A	1570	1·30	42	11·1	0·01	5·5	71·9

Limiting assumptions

The temperature field is described by equations (16) and (19). This model does not, however, take the recirculation of gases into consideration. Due to recirculation, hot gases are brought back to the burner end of the furnace and equation (19) gives values of gas temperature which are too low around the jet close to the burner. Here, the limiting assumption, that the gas temperature of the exit of the furnace is minimum gas temperature all over the furnace, will be a sufficiently good approximation.

As is seen from Fig. 6, the measured values of K tend to approach asymptotically a value of about 0·001 mm^{-1}. This corresponds to gas radiation from pure gases of the temperature (900–700°C) and composition (12–14% CO_2 and H_2O) in question and a radiation path length of the measuring instrument; the soot particles have burnt out. This is in accordance with Hottels charts for total gas radiation [13] where

$$\varepsilon_{gD} = 1 - e^{-KD}$$

ε_{gD} is the gas emissivity according to [13] for a gas layer of $D = 45$ mm which was used in the measurements. Thus $K > 0{\cdot}001$ means predominantly grey flame radiation and $K \simeq 0{\cdot}001$ means gas radiation for a layer of the thickness of D. As the mean beam length in the gas filled parts of the furnace is much longer than D, $K = 0{\cdot}001$ will be too large and an average mean beam length corresponding to $K = 0{\cdot}00034$ mm^{-1} for the very irregular gas volumes surrounding the luminous flame will be assumed. $K = 0{\cdot}00034$ is in this case the minimum value of the K-field calculated with equations (17) and (20).

This is an important approximation in the calculation of gas radiation which has to be maintained until a wavelength-dependent calculation can be done within a reasonable time of computation.

RESULTS

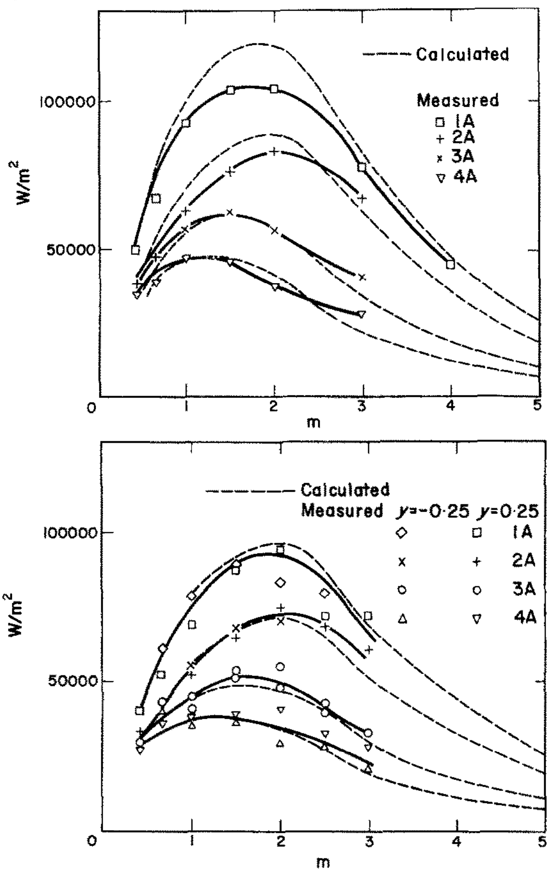
The total amount of heat received by the walls of the combustion chamber (as measured by the heat-flux meters, calibrated for incident radiation) is estimated by a graphical integration and compared with the total amount of heat absorbed by the walls (calculated knowing the mass flow and temperature rise of the cooling water of the furnace). This comparison gives an effective "absorptivity" of the walls of 0·80 (the four trials are within ±2·5 per cent of this value).

In this "absorptivity" the influence on the heat transfer by the thin layer of soot deposited on the walls is also included. An approximative calculation of the effect of the layer of soot indicates that in this case (thickness 0·5–1·0 mm) the actual absorptivity is close to 0·9 which is a value commonly used in heat-transfer calculations of combustion chambers, e.g. [13]. However, in the case of the grey radiation approximation the re-radiated flux from the surface of the layer of soot may be considered as reflected radiation and this will be done below, where the reflected radiation will be composed by re-radiation and reflexion from the walls corresponding to the "absorptivity" of 0·80.

Thus reflected radiation might play an essential role in the transfer of heat in the furnace particularly close to corners where the walls are exposed to each other and not shadowed by absorbing material of the flame. The temperature field, however, is not influenced, as it is known *a priori*.

The calculation of heat flux is carried out in two steps. First the contribution to the wall heat flux from the flame is calculated by means of equations (8), (13) and (14). The additional contribution to the heat flux by convection from the gases is estimated using the conventional methods and is found to be small (in the order of 2 per cent) compared to heat flux by radiation.

The reflected flux, leaving the surfaces, is estimated to 20 per cent of the calculated heat flux from flame and convection knowing the wall absorptivity. The first summation part of equation (8) and the equation (13) now gives the heat flux on the walls due to reflection in the neighbouring walls and absorption in flame and gases in between. This calculation should be repeated with the new heat flux obtained, and so on. This would mean a simulation of the integral equation of radiation (4) but one reflection gives a sufficiently good estimation, as the reflected parts of the total heat flux on the wall is only 5–7 per cent of the total incident heat flux at $y = 0$. As assumed, the points at $y = \pm 0{\cdot}25$ on the side wall receive more than the points at $y = 0$.



FIGS. 10 and 11. Comparison between calculated and measured heat-flux on the wall.

Figures 10 and 11 show a comparison between calculated heat flux (from the flame, from the refractory surface, from reflection on the neighbouring walls and due to convection) and the measured values.

It is seen that the main deviation of 15 per cent is at trial 1A. A review of the sources of error leads to the conclusion that the main reason for the differences, if possible errors of measurement are excluded, is the evident fact that the mathematical model is only an approximation of a rather irregular turbulent flame.

DISCUSSION AND CONCLUSION

It is most probable that the errors due to the simplification of the theoretical considerations are of a negligible importance in the class of heat transfer problems treated here. In many cases even the black-wall approximation is sufficient, for instance, for parts of walls situated far from neighbouring surfaces.

The main difficulty lies in describing the flame characteristics using a simple mathematical model, and the results are dependent on the accuracy of the model. In a practical case it is of course not possible to define the temperature and K-fields as closely as have been done here by measurements, but when the geometrical arrangement and burner design are known, the general picture of the mathematical expressions is given and a few measurements or estimates will be sufficient to define a model. By changing the model or the geometry of the furnace, investigations of heat transfer can be carried out which are considered necessary for the construction of the furnace.

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REFERENCES

1. P. K. KONAKOV, S. S. FILIMONOV and B. A. KHRUSTALEV, Heat transfer in steam boiler combustion chambers, *Izdatel'stvo Rechnoi Transport*, Moscow (1960).
2. A. M. GURVICH, *Standard Method for Heat Calculation of Furnaces* edited by A. M. GURVICH and N. V. KUZNETSOV. Gosenergoizdat, Moscow (1958).
3. C. H. YLAND, Application of the similarity theory on the radiation in furnaces, CTH-handl. 135, (Trans. of Chalmers Univ. of Technol.), Gothenburg (1953).
4. J. M. BEER, The significance of modelling, *J. Inst. Fuel* **39**, 466–473 (1966).

5. A. M. GODRIDGE, Heat transfer in the furnace chamber of pulverized-fuel-fired water-tube boilers, *J. Inst. Fuel* **40** 300–309 (1967).
6. A. CRAYA and R. CURTET, Sur l'évolution d'un jet en espace confiné, *C.r. Lebd. Séanc. Acad. Sci., Paris* **241**, 621–622 (1955).
7. M. W. THRING, *The Sciences of Flames and Furnaces*, p. 446. Chapman & Hall, London (1962).
8. H. C. HOTTEL and E. S. COHEN, Radiant heat exchange in a gas-filled enclosure: allowance for nonuniformity of gas temperature, *A.I.Ch.E. Jl* **4**, 3–14 (1958).
9. H. C. HOTTEL and A. F. SAROFIM, The effect of gas flow patterns on radiative transfer in cylindrical furnaces, *Int. J. Heat Mass Transfer* **8**, 1153–1169 (1965).
10. R. GÜNTHER and S. HERING, Mathematisches Modell zur Berechnung der Wärmeabgabe von nichtleuchtenden Strahlflammen in Feürräumen, *Brennst.-Wärme-Kraft* **20**, 200–205 (1968).
11. M. W. THRING, P. J. FOSTER, I. A. MCGRATH and J. S. ASHTON, Prediction of the emissivity of hydrocarbon flames, *Conf. Int. Dev. Heat Transfer*. Part IV, pp. 796–803, Mech. Engrs., London and ASME (1961).
12. V. R. STULL and G. N. PLASS, Emissivity of dispersed carbon particles, *J. Opt. Soc. Am.* **50**, 121–128 (1960).
13. H. C. HOTTEL, Radiant heat transmission, in *Heat Transmission* edited by W. H. MCADAMS. McGraw-Hill, New York (1954).
14. J. O. HINZE and B. G. v. d. HEGGE ZIJNEN, Transfer of heat and matter in the turbulent mixing zone of an axially symmetrical jet, Delft Publication 100 (1948).
15. G. ABRAMOVICH, *The Theory of Turbulent Jets*. Cambridge, Mass. (1963).

RAYONNEMENT A PARTIR DE FLAMMES ET DE GAZ DANS UNE CHAMBRE DE COMBUSTION A PAROI FROIDE

Résumé—Le calcul du transport de chaleur dans les foyers est habituellement basé sur des approximations grossières de la température et de l'émissivité dans la chambre de combustion. Cet article suggère une méthode convenable pour calcul à la machine du transport de chaleur à partir de flammes et de gaz vers les parois d'un foyer. Une démonstration théorique de l'équation de base est faite et les simplifications nécessaires pour l'éxécution du calcul dans un temps acceptable sont discutées. Les distributions de température et d'émissivité de la flamme sont décrites au moyen de fonctions simples, comme modèle mathématique. Des expériences ont été effectuées dans un foyer de forme simple en employant un brûleur à pétrole afin de vérifier la précision de hypothèses théoriques. Les mesures des températures de la flamme et due gaz, de l'émissivité de la flamme, du flux de chaleur pariétal ont été faites ainsi que des mesures extérieures nécessaires pour le bilan de chaleur du foyer. Les mesures de la température et de l'émissivité définissent le modèle mathématique.

Le transport de chaleur à partir de la flamme et des gaz (en y incluant les contributions dues à la réflexion, aux surfaces non froides et à la convection) a été calculé pour quatre charges différentes du foyer et comparé aux mesures du flux chaleur. La différence entre les flux de chaleur calculés et mesurés, due aux erreurs de mesure et de cacul, était inférieure à environ 15 pour cent du flux de chaleur total.

STRAHLUNG VON FLAMMEN UND GASEN IN EINER KALTWAND-VERBRENNUNGSKAMMER

Zusammenfassung—Die Berechnung des Wärmeübergangs in Heinzkesseln beruht üblicherweise auf groben Schätzungen der Temperaturen und der Emissionsverhältnisse in der Brennkammer. In dieser Arbeit wird ein Verfahren zur numerischen Berechnung des Wärmeübergangs von der Flammen und Gasen an die Wände des Ofens dargelegt. Die grundlegenden Gleichungen und die Vereinfachungen, die für eine erträgliche Rechenzeit notwendig sind, werden abgeleitet. Der Verlauf der Temperaturen und der Emissionsverhältnisse der Flammen werden mit einfachen Funktionen beschrieben. An einem Versuchsofen einfacher Geometrie, geheizt mit einem Ölbrenner, wurden Versuche durchgeführt um die Genauigkeit der theoretischen Annahmen zu prüfen. Gemessen wurden die Flammen- und Gastemperaturen, Emissionsverhältnis der Flammen, Wärmestrom an der Wand, ferner Messungen ausserhalb für die Wärmebilanz des Ofens. Die Messungen der Temperaturen und der Emissionsverhältnisse bestimmen das mathematische Modell.

Der Wärmeübergang von Flammen und Gasen (einschliesslich des Beitrags der Reflexion, nicht-kalter Oberflächen und der Konvektion) wurde für vier verschiedene Belastungen des Ofens berechnet und mit den gemessenen Wärmeströmen verglichen. Der Unterschied zwischen berechneten und gemessenen Wärmeströmen infolge der Fehler der Messungen und der Rechnung lag unter 14 Prozent des gesamten Wärmestroms.

ИЗДУЧЕНИЕ ОТ ПЛАМЕНИ И ГАЗОВ В КАМЕРЕ СГОРАНИЯ С ХОЛОДНЫМИ СТЕНКАМИ

Аннотация—Расчет теплообмена в печах обычно базируется на грубых приближениях о температуре и излучательной способности в камере сгорания. В данной статье предлагается метод, удобный для машинного расчета теплообмена от пламен и газов к стенкам печи. Теоретически выводится основное уравнение, а также рассматриваются упрощения, необходимые для выполнения расчетов. С помощью простых функций и математической моделп описывается температурное поле и излучательная способность пламени. Чтобы проверить точность теоретических допущений, проводились эксперименты в экспериментальной печи простой конфигурации с одной горелкой. Измерялись температуры пламени и газа, излучательная способность пламени, тепловой поток на стенке, внешняя область, что необхолимо для теплового баланса печи. Измерения температуры и излучательной способности явились основой для создания математической модели.

Теплообмен пламени и газов (с учетом влияния отражения, нагретых поверхностей и конвекции) рассчитывался для четырех различных нагрузок печи и сравнивался с измерениями теплового потока. Различие между измеренным и рассчитанным тепловым потоком за счет погрешности измерений и расчетов примерно на 15% меньше общего теплового потока.